Optimal Automatic Multipass Shader Partitioning by Dynamic Programming

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Disclaimer

This talk describes GPU architecture research carried out at Sony Computer Entertainment.

It does not describe any commercial product.

In particular, this talk does not discuss the PLAYSTATION 3 nor the RSX.

Outline

The problem:

- Automatically compile large shaders for small GPUs.

The insight:

- This is a classical job-shop scheduling problem.

The proposed solution:

- Dynamic Programming.

Outline++

The problem:

- Automatically compile large shaders for small GPUs.
  - Exhaust registers, interpolants, pending texture requests, ...
  - Goal: optimal solutions, scalable algorithm.

The insight:

- This is a classical job-shop scheduling problem.
  - Job-shop scheduling is NP-hard/complete.
  - *Well-studied problem, many solution algorithms exist.*

The proposed solution:

- Dynamic Programming.
  - Satisfies nonlinear objective function.
  - Optimal and (semi-)scalable.

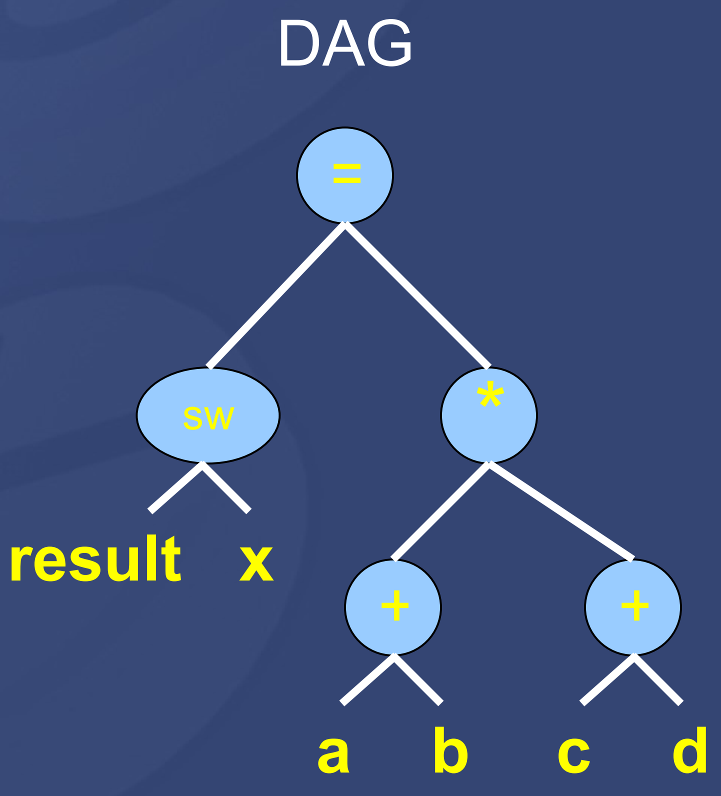
The Problem

Physical resources are limited.

- Rasterized interpolants.
- GP registers.
- Pending texture requests.
- Instruction count.
- etcetera

A very simple example:

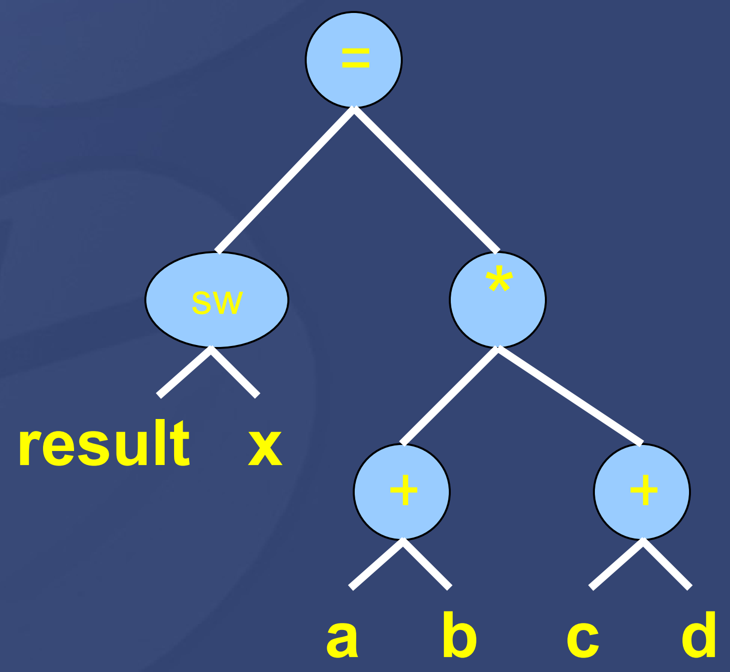
- result.x = (a+b)*(c+d)
- Requires three GP registers
  - Multiple passes with two registers



result.x = (a+b)*(c+d)

R0	R1	
a		Load R0=a
a	b	Load R1=b
a+b	b	R0=+(R0,R1)
a+b	c	Load R1=c
a+b	c	Store R0 → aux
d	c	Load R0=d
c+d	c	R0=+(R0,R1)
c+d	a+b	***New Pass***
(a+b)*(c+d)	a+b	R0=*(R0,R1)
(a+b)*(c+d)	a+b	Store R0 → swizzle(result,x)

DAG



The MPP Problem

Multipass Partitioning Problem [Chan 2002]

Given:
- An input DAG.
- A GPU architecture.

Find:
- A schedule of DAG operations.
- A partition of that schedule into passes.

Such that:
- Schedule observes DAG precedence relations.
- Schedule respects GPU resource limits.
- Runtime of compiled shader is minimal *(optimality).*
  - *(Chan: number of passes is minimal.)*

References

Graphics Hardware 2002:

- *Efficient partitioning of fragment shaders for multipass rendering on programmable graphics hardware.*
  E. Chan, R. Ng, P. Sen, K. Proudfoot, P. Hanrahan

Graphics Hardware 2004:

- *Efficient partitioning of fragment shaders for multiple-output hardware.*
  T. Foley, M. Houston, P. Hanrahan
- *Mio: fast multipass partitioning via priority-based instruction scheduling.*
  A. Riffel, A. Lefohn, K. Vidimce, M. Leone, J. Owens

Requirements: Optimal, Scalable

Nonlinear cost function.

- Depends on current machine state.

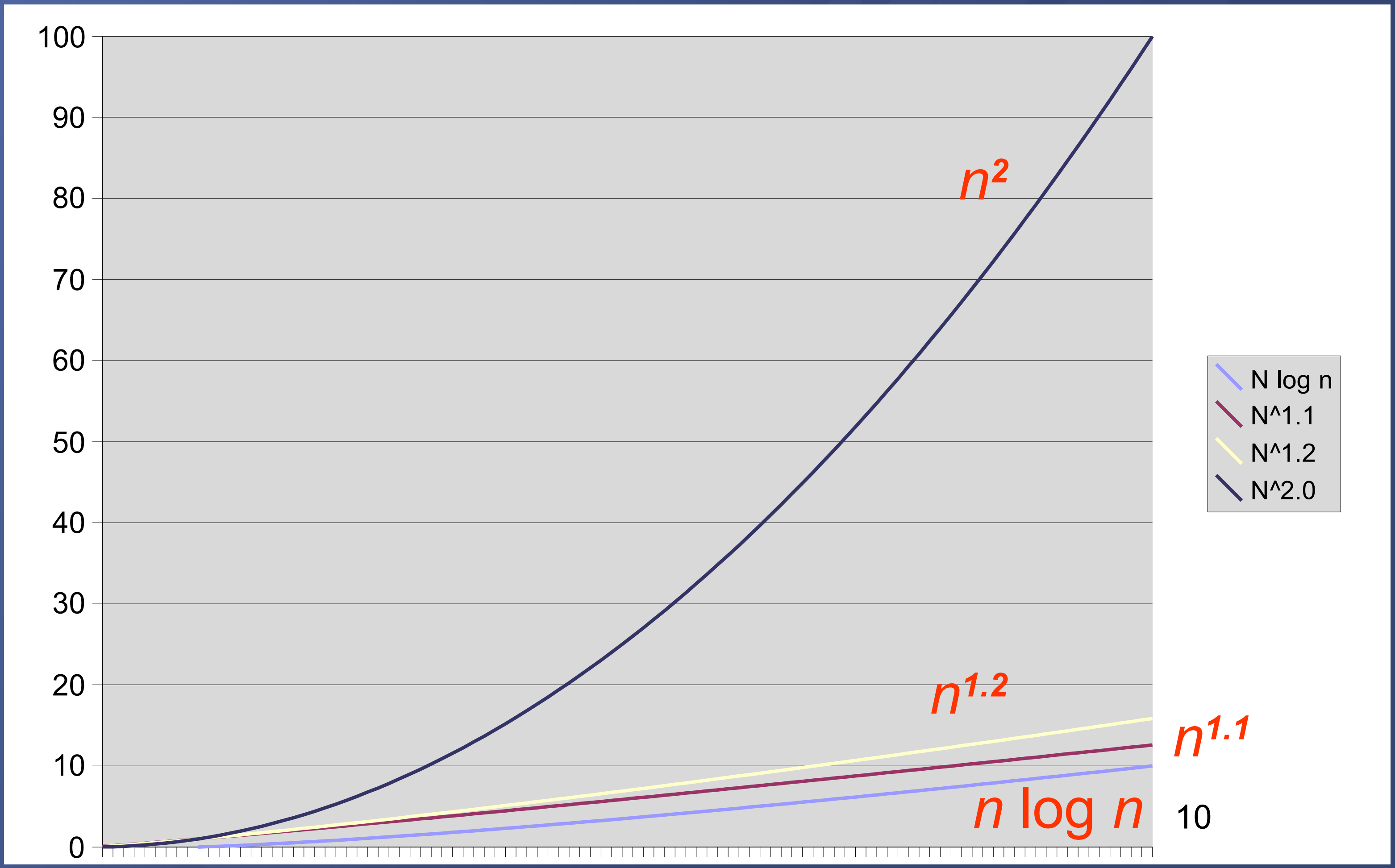
Optimal solutions:

- (Many) fine-grained passes.
- Long shaders.
  - High-dimensional solution space.
  - Many local minima (suboptimal solutions).

Scalable algorithm:

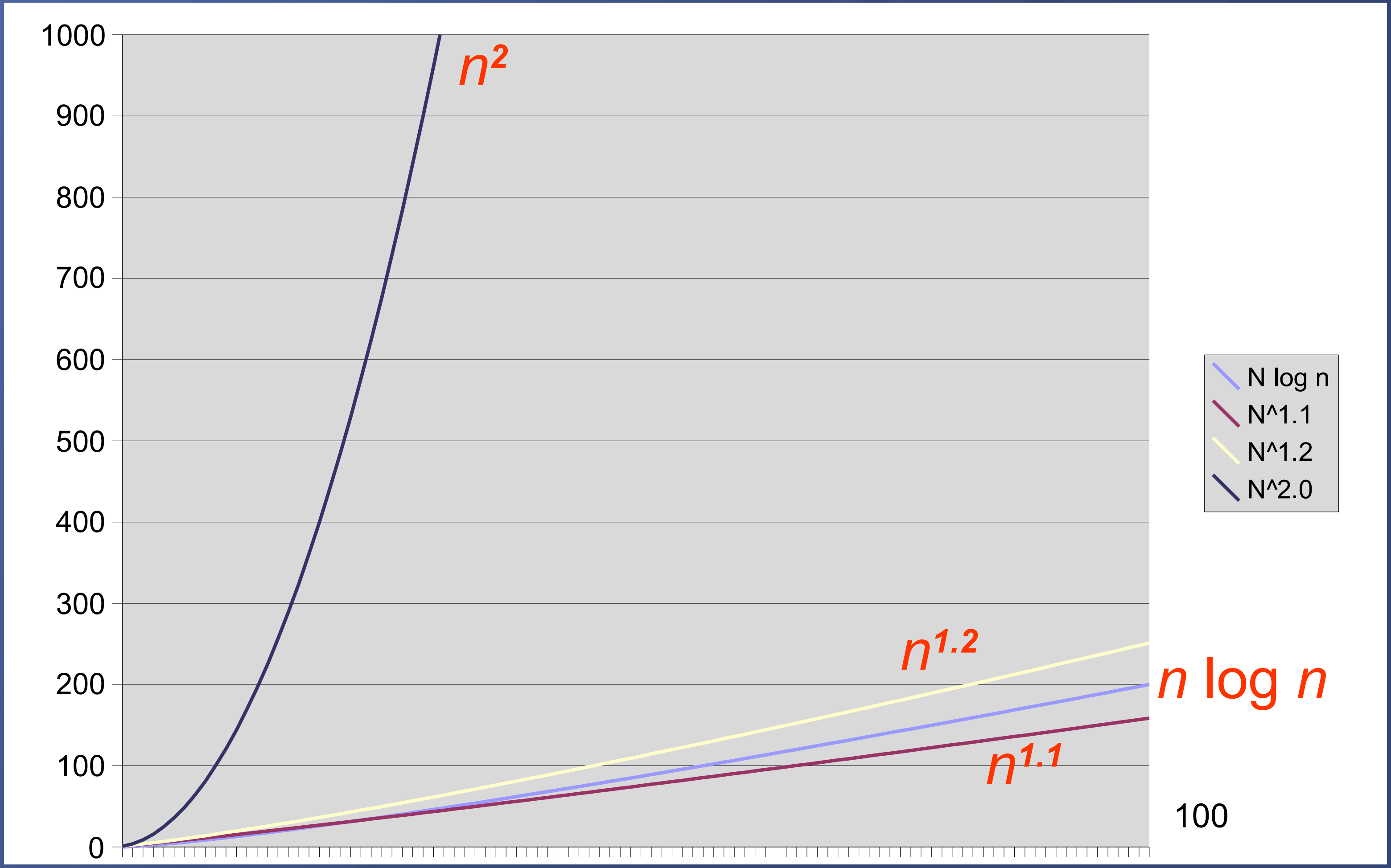
- Compile-time cost must not grow unreasonably.
  - $O(n \log n)$ is scalable.
  - $O(n^2)$ is not scalable.

Scalability, n=10



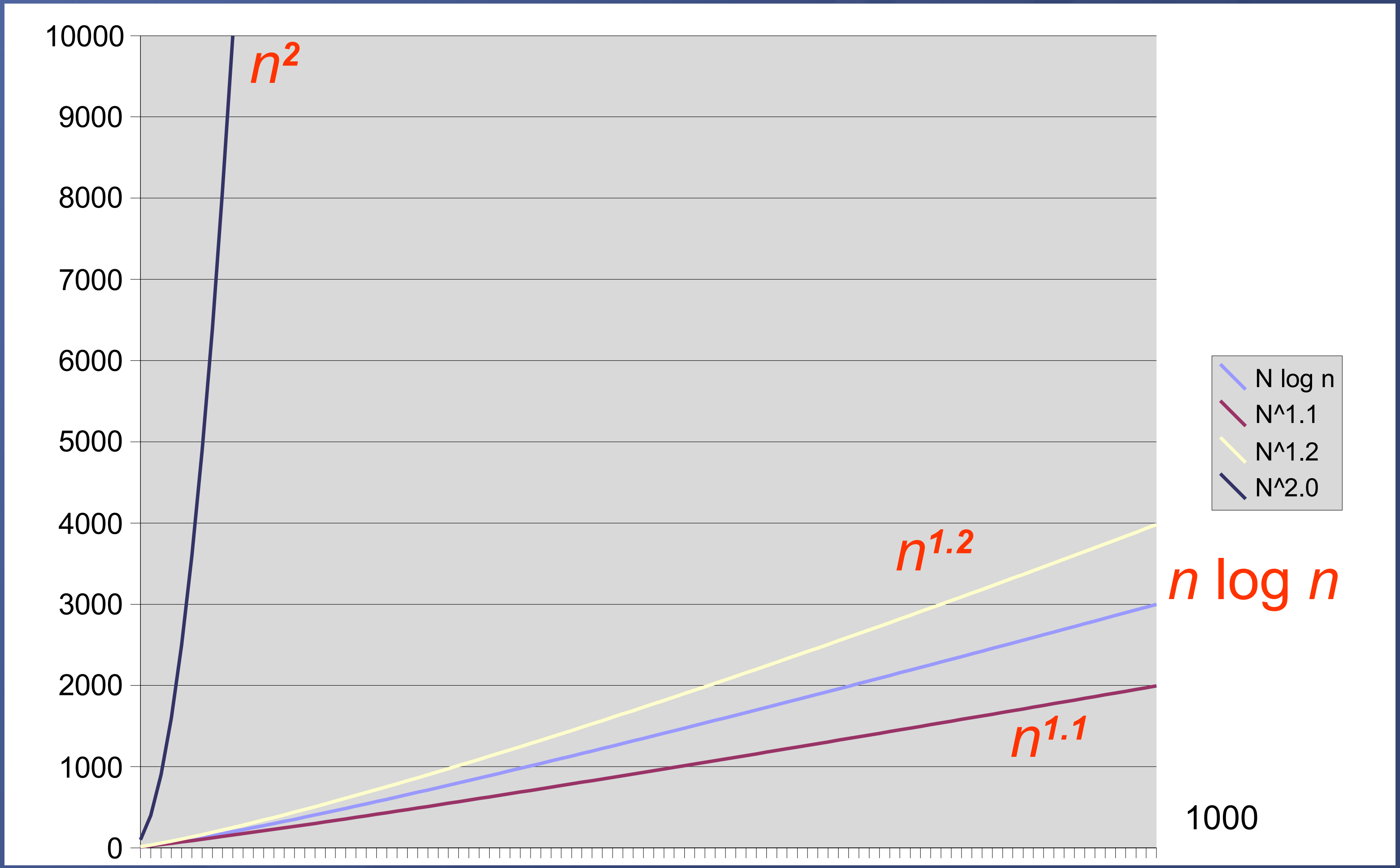
Scalability, n=100

(Current vertex shaders)



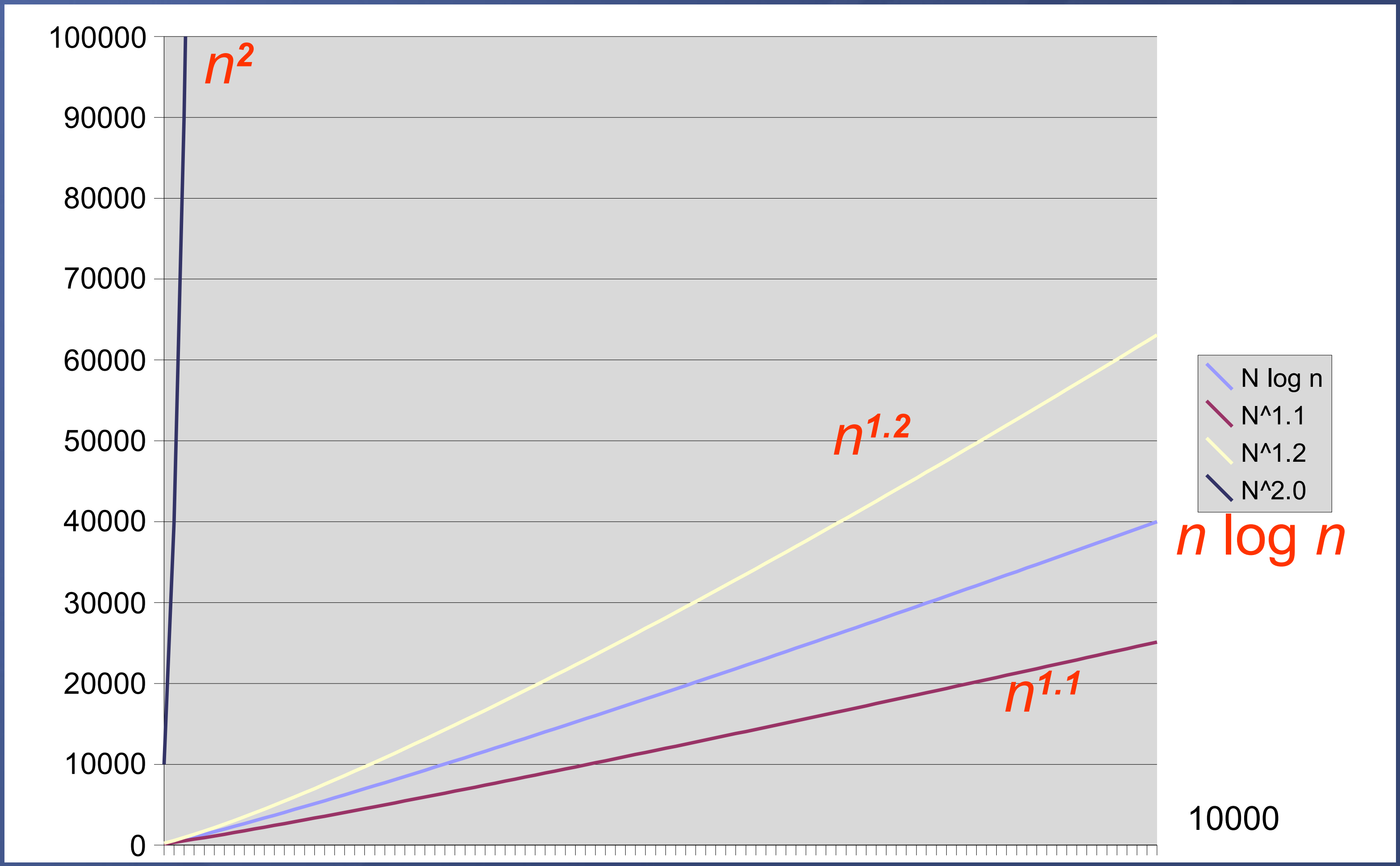
Scalability, n=1000

(Current real-time fragment shaders)



Scalability, n=10000

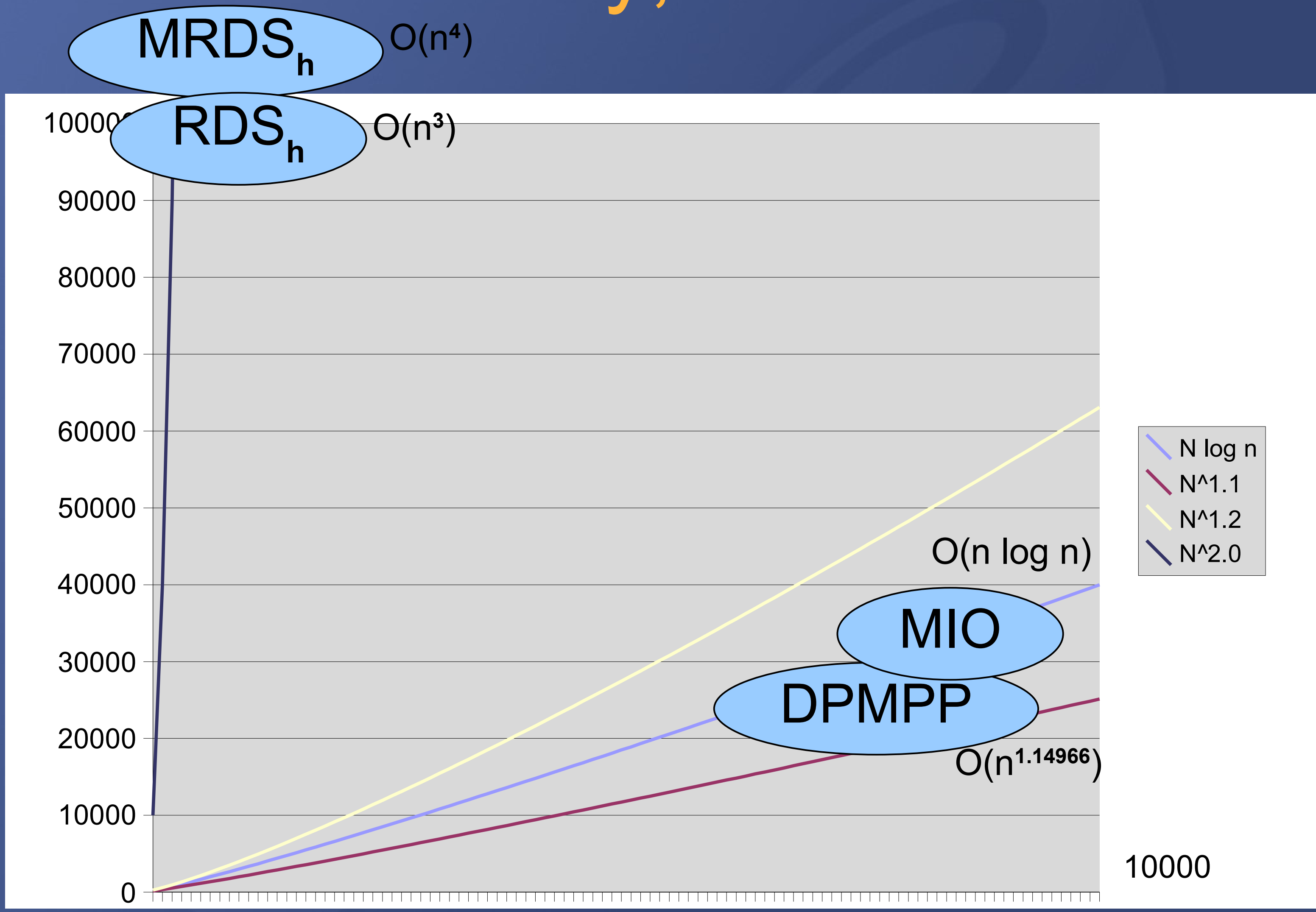
(Current GPGPU fragment shaders)



Three Proposed Solutions

- Minimum cut sets (RDS_h, $MRDS_h$) [Chan 2002, Foley 2004]
  - Graph (DAG) cut sets.
  - Minimize number of cuts.
  - Greedy algorithms.
  - $O(n^3)$, $O(n^4)$, nonscalable.
- List scheduling (MIO) [Riffel 2004]
  - Job scheduling.
  - Minimize instruction count (linear function).
  - Greedy algorithm.
  - $O(n \log n)$, scalable.
- Dynamic programming (DPMPP) [this paper]
  - Job scheduling.
  - Minimize predicted run time (nonlinear function).
  - Globally optimal algorithm.
  - $O(n^{1.14966})$ empirically, (semi-) scalable.

Scalability, n=10000



The Insight: Job Shop Scheduling

An NP-hard multi-stage decision problem.

- A set of time slots and functional units.
- A set of tasks.
- An objective function (cost).

Goal: assign tasks to slots/units to minimize cost.

Examples:

- Compiler instruction selection.
- Airline crew scheduling.
- Factory production planning.
- etcetera

Solving project scheduling problems by minimum cut computations. R. Mohring, A. Schulz, F. Stork, M. Uetz. Management Science (2002), pp. 330-350.

Job Shop Scheduling for MPP

Defined by DAG and GPU architecture.

- A set of n DAG operations (+ “new pass” operation).
- A schedule with n time slots.
- A single GPU processor.
- Cost function predicts quality of compiled code.
  - Predicted execution time (DPMPP).
  - Instruction count (MIO).
  - Number of passes (RDS_h, $MRDS_h$).

Many possible formulations and solution algorithms.

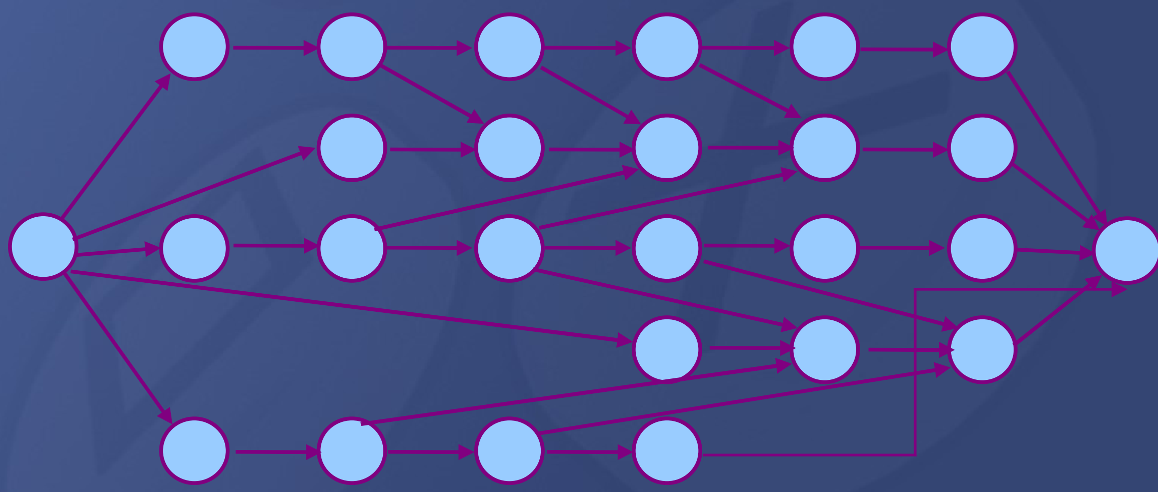
- Integer programming, linear programming, dynamic programming, list scheduling, graph cutting, branch and bound, Satisfiability, ...
- Problem size can often be $O(n^2)$ [nonscalable]

Integer Programming Formulation

- Jobs (tasks) *j, times t; unknowns x.*
- 0-1 decision variables $x_{j,t}$=1 iff job *j* is scheduled at time *t.*
- Costs $w_{j,t}$ = time-dependent cost of job *j* at time *t.*
- Resource requirements $r_{j,k}$ for job *j* of resource *k.*
- Constraints:
  - Precedence: $\Sigma_t\, t(x_{j,t} - x_{i,t}) \quad >= \quad d_{i,j}$
  - Resource: $\Sigma_t\, r_{j,k}(\Sigma^t_{s=t-pj+1}\, x_{j,s}) <= R_k$
  - Uniqueness: $\Sigma_t\, x_{j,t} \quad = \quad 1$
- Objective:
  - Minimize $\Sigma_{j,t}\, w_{j,t} x_{j,t}$ subject to constraints (linear objective).
- Various solvers (simplex, Karmarkar's algorithm, ...).
  - Potentially exponential worst-case time.
  - Easy transformation to SAT (Boolean decision variables).
    - Different solvers (CHAFF, branch and bound, Tabu, ...)
- $\| X \|$ is $O(n^2)$.

Graph Cut Formulation

- See [Mohring 2002] for details.
- Vertices $v_{j,t}$ represents job *j* scheduled at time *t.*
- $v_{j,first(j)}$ ... $v_{j,last(j)}$ *marks all possible times for job j.*
- *Temporal arcs* ($v_{i,t}$, $v_{j,t+d_{ij}}$) *for time lags* $d_{i,j}$ *have infinite capacity.*
- *Assignment arcs* ($v_{j,first(j)}$, $v_{j,first(j)+1}$) *have capacity* $w_{j,t}$.

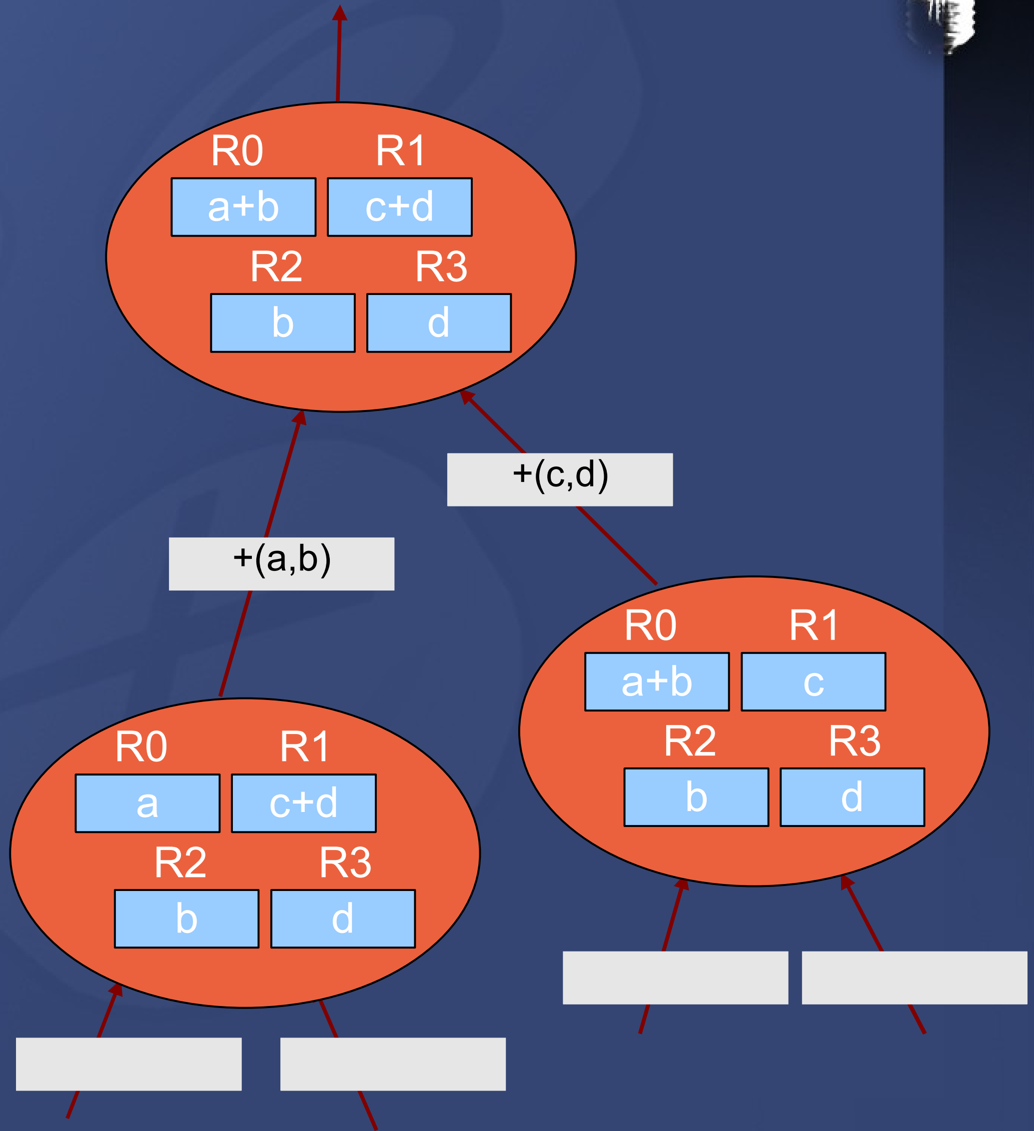


- A minimum cut in this graph defines a minimum cost schedule.
- O(*m* log *m*) time for *m* vertices [but *m is* O(n^2)].

Dynamic Programming Formulation

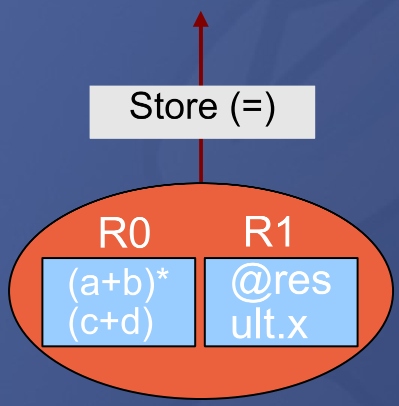
Search tree root is terminal end state at time t=n.

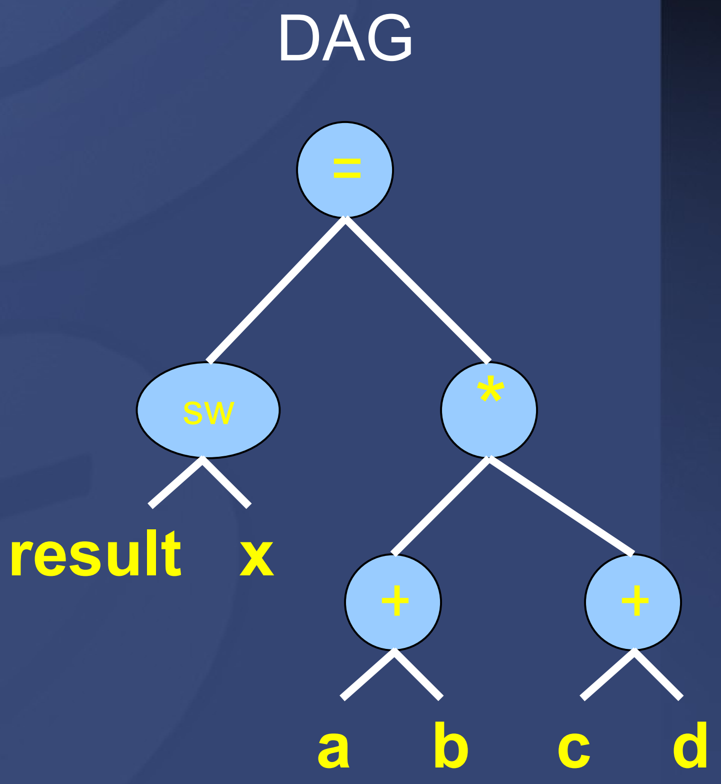
- Vertices are snapshots of machine state.
- Edges are transitions (DAG operation, or "new pass").
  - Generate tree breadth-first.
  - Leaves represent initial states (time t=1).
  - Every path from leaf to root is a valid schedule.
- MPP solution is the lowest-cost path.
  - Time and space are O(n^b) where *b* is the average branching factor.
- Prune maximally.
- (*b* < 1.2) ⇨ (semi-)scalable.



Dynamic Programming Example

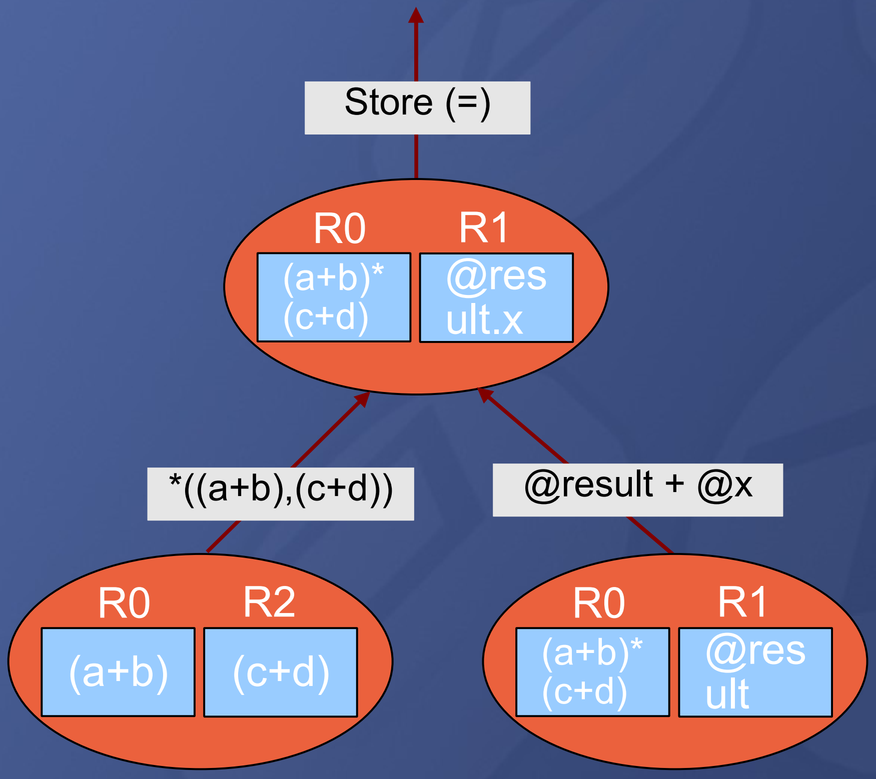
- Root is terminal end state (time *t=n*).

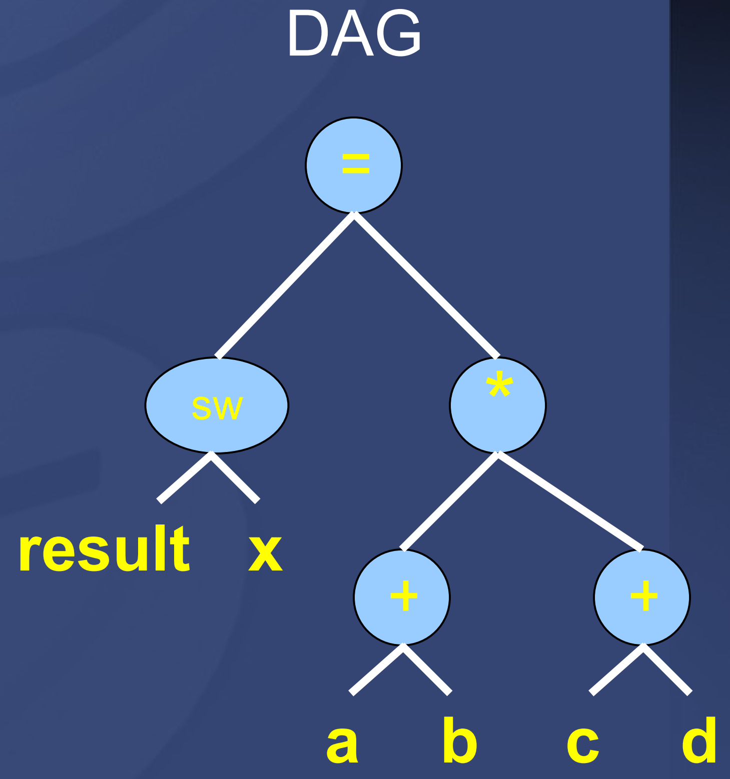




Dynamic Programming Example

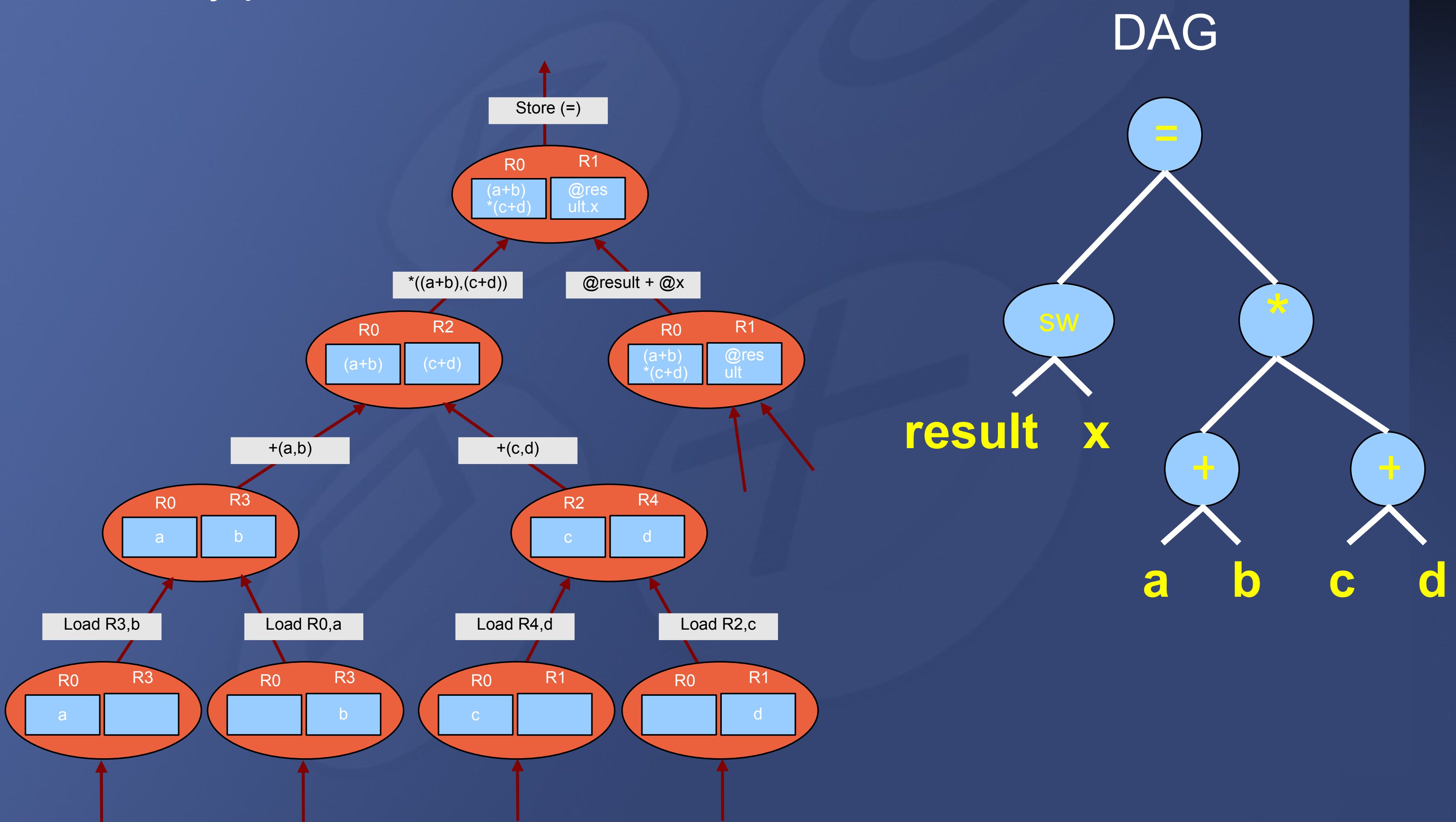
- Generate tree breadth-first.
  Accumulate cost along paths.





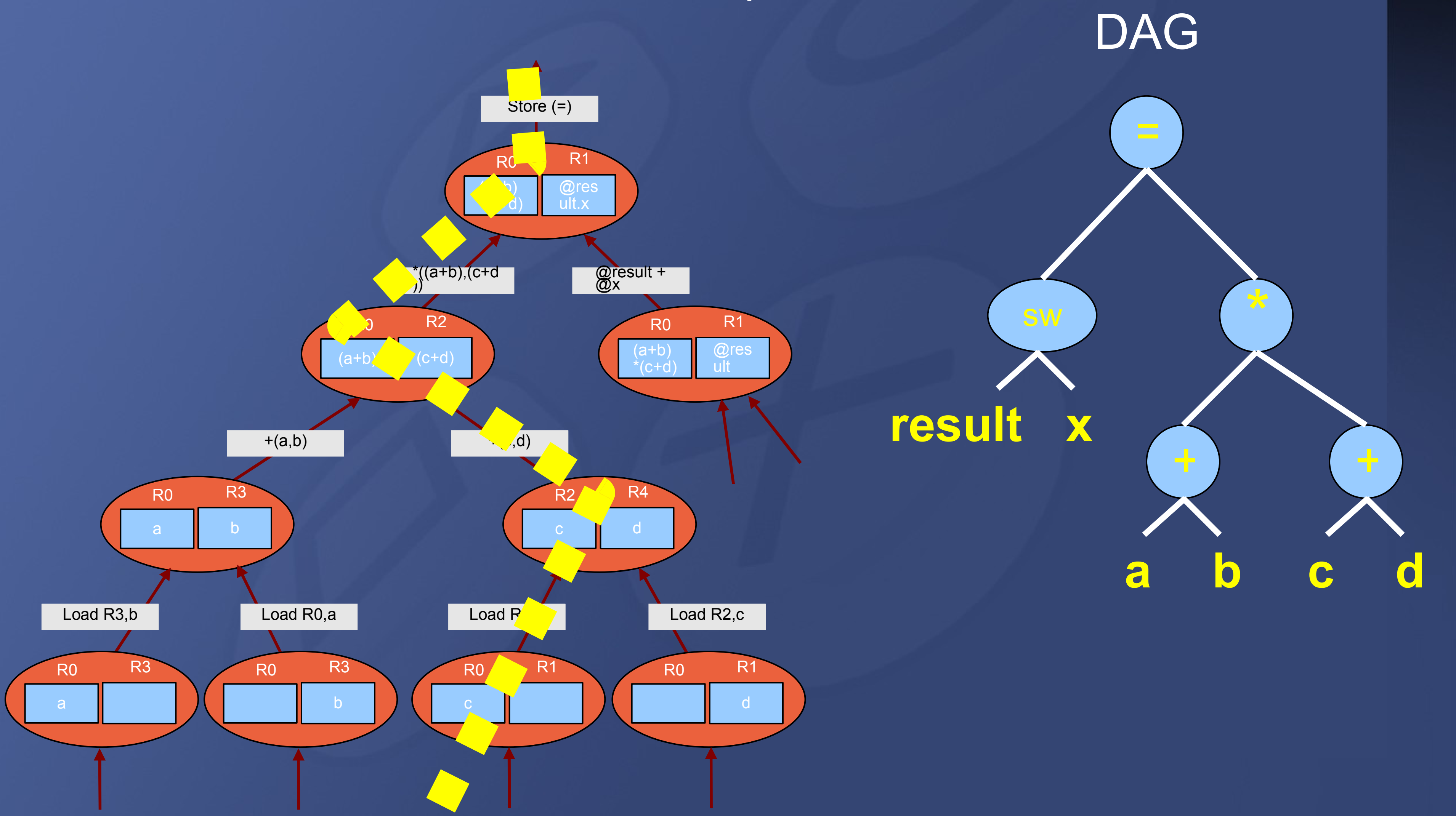
Dynamic Programming Example

- Every path from leaf to root is a valid schedule.



Dynamic Programming Example

- MPP solution is the lowest-cost path.



Key Elements of DP Solution

Solve problem in reverse.
- Start from optimal end state.
- Requires Markov property.

- Prune maximally.
  - Manage complexity.
  - “optimal substructure”.

Retain all useful intermediate states.

Consider all valid paths to find solution.

Markov property

(Stale operands)

R0	R1	
a		Load R0=a
a	b	Load R1=b
a+b	b	R0=+(R0,R1)
a+b	c	Load R1=c
a+b	c	Store R0
d	c	Load R0=d
c+d	c	R0=+(R0,R1)
c+d	a+b	*New Pass*
(a+b)*(c+d)	a+b	R0=*(R0,R1)
(a+b)*(c+d)	a+b	Store R0

R0	R1	
d		Load R0=d
d	c	Load R1=c
c+d	c	R0=+(R0,R1)
c+d	b	Load R1=b
c+d	b	Store R0
a	b	Load R0=a
a+b	b	R0=+(R0,R1)
a+b	c+d	*New Pass*
(a+b)*(c+d)	c+d	R0=*(R0,R1)
(a+b)*(c+d)	c+d	Store R0

Markov property holds for ...

- GP registers
- Rasterized interpolants
- Pending texture requests
- Instruction storage
- etcetera

Nonlinearity and Optimality

Algorithm	Objective	Linearity
RDS_h, $MRDS_h$	*passes*	*linear*
MIO	*instructions*	*linear*
DPMPP	*execution time*	*nonlinear*

GPU cost function can be nonlinear.

- Depends on current machine state.
  - *E.g. pipelined activity due to previous operation.*
- COST($instr_A$)+COST($instr_B$) <> COST($instr_A$,$instr_B$)
  - *Linear objective functions are approximations to reality (e.g. instruction count).*

Nonlinear functions can have many minima.

- Functions for real GPUs are ugly.
- Greedy algorithms become trapped in local minima.
  - Dynamic Programming computes global minima.
  - Dynamic Programming solutions are globally optimal.

Optimal substructure

DP algorithms must avoid search tree explosion.

- Complexity $O(n^b)$, average branching factor b.
- Need to prune search space.
- Strong preference for local branching factor 1 (scalability).

Optimal substructure:

- Compatible with global solution (conservative evaluation).
- Can evaluate locally.

Objective:

- Minimize predicted execution time.
- Approximate by minimizing number of register loads.
  - Locally computable, globally conservative (includes solution).

Implementation:

- Schedule shortest DAG subtree (DPMPP).
  - Schedule is generated in reverse order.
  - Result is ordered largest to smallest.

Algorithm DPMPP

Stage is initially equal to n, and recurses down to 1.

T is initially the set of final transitions to the end state. Subsequently, T is P from the previous stage.

P is the current set of transitions being explored (search tree cross-section)

S_t is the set of locally optimal transitions that could be scheduled before this *t*.

Cost $DPMPP(stage, T)$ {
  $P := \{\}$
  $(\forall t \in T)$ do
    $S_t := \left\{ \begin{array}{l} \text{valid prior transitions of } t \\ \text{with optimal substructure} \end{array} \right\}$
    $min := \infty$
    $(\forall s \in S_t)$ do
      $s.cost := t.cost +$
        $COST(s.operation, s.postcondition)$
      if $(s.cost < min)$
        $min := s.cost$
    enddo s
    $(\forall s \in S_t$ such that $s.cost = min)$ do
      $P := (P \cup s)$ uniquely by precondition
    enddo s
  enddo t
  if $(stage > 1)$
    return $DPMPP(stage - 1, P)$
  else
    return $\min_{p \in P}(p.cost)$ }

Figure 1

Choose shortest subtrees or reverse Sethi-Ullman numbering.

Cost is computed with respect to s.precondition.

Only keep minimum cost path(s) for this *t*.

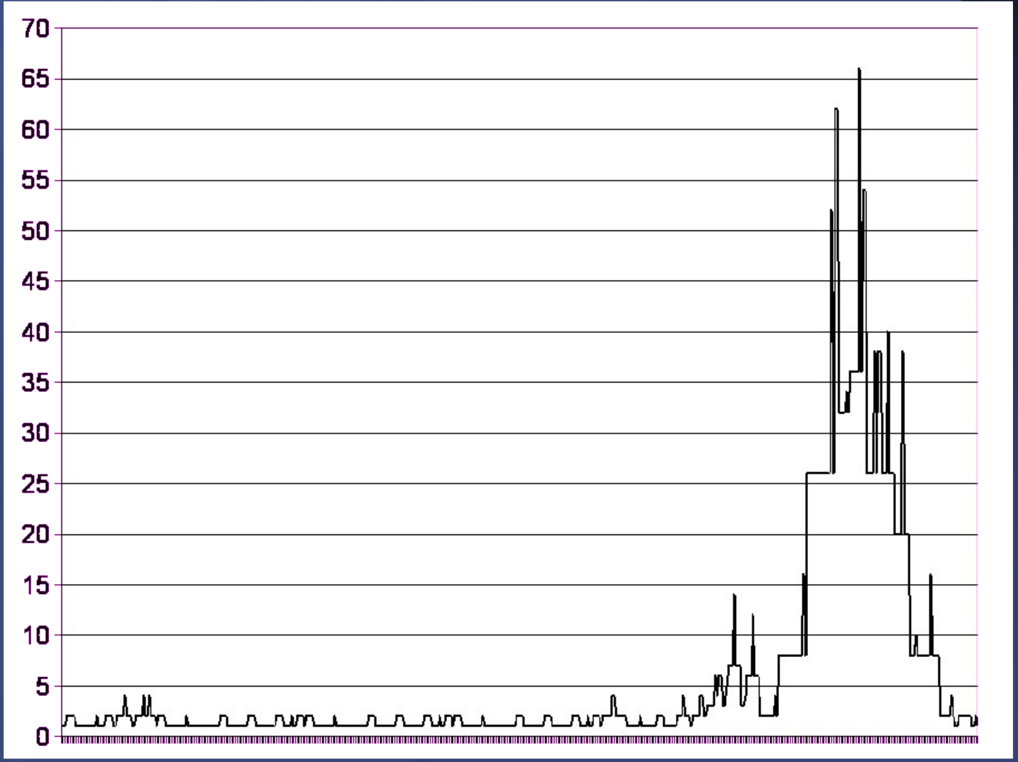
Also discard redundant *s.precondition.*

Continue the breadth-first search of the tree.

Terminate when Stage = 1. The global solution is a path from the cheapest *p* in *P* to the search tree root.

Scalability

- Search width w_i.
- Branching factor $b_i = w_{i+1}/w_i$.
- Area under the curve is equal to n^b where b is the average b_i.
- Observation: b decreased with increasing n.
  - Is this a pattern?
  - Requires that area grow less than unit for each unit increase in n.
- Implication: asymptotic scalability.
  - Don't know.



Search tree width over n=490 stages.
(Real-time fragment shader, b=1.06091).
Figure 2

Optimal Substructure Revisited

Sethi-Ullman numbering.

- Orders DAG nodes by number of subtree register usage.
- Used in algorithm MIO to prioritize operations.
  - Highest numbered nodes first.
  - Schedule generated in order.
- Should be explored for dynamic programming.

Subtree size.

- Monotonic in subtree register usage.
- Used in algorithm DPMPP to prioritize operations.
  - Smallest numbered nodes first.
  - Schedule generated in reverse order.
- Probably less accurate than Sethi-Ullman.
  - Implication: less efficient compilation.

DPMPP and MIO

	MIO	DPMPP
	List Scheduling	DP
Direction:	*forward*	*reverse*
Paradigm:	*greedy*	*optimization*
Optimality:	*local*	*global*
Complexity:	$\mathcal{O}(n \log n)$	$\mathcal{O}(n^{1.14966})$
Numbering:	*Sethi – Ullman*	*subtree size*
Critical:	*scheduling priority*	*search pruning*
Policy:	*largest trees first*	*smallest trees last*
Affects:	*run time*	*compile&runtime*

Figure 4

Conclusions

Claims:

- Nonlinear cost functions are required for accuracy.
- Algorithm DPMPP:
  - Is GPU-generic.
  - Supports nonlinear cost functions.
  - Finds globally optimal solutions.
  - Is scalable above $n=10^5$.
  - May be asymptotically scalable.

Remarks:

- DPMPP should use Sethi-Ullman numbering.
- Shader multipassing provides diverse benefits.
  - Some benefits require accurate (i.e. detailed) cost functions.
- Primary challenge is inter-pass data transfer.
  - Challenge: zero (effective) latency transfer mechanisms.
    - e.g. F-Buffer with zero latency.
    - Simpler solutions are possible.